COMP 761: Mathematical Tools for CS

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September 2, 2020

Welcome to COMP-761

This is a whirlwind introduction to important math that turns up everywhere in computer science. We will explore many topics. But the goal is really to learn how to reason mathematically about problems in computer science from the ground up – and how to prove statements formally when you need to.

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No background in theoretical math is necessary

Introductions

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2. Proofs and how to think about problems

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Note: An exception may be made for the attendance/participation portions of the grade for those students who are attending the class remotely from a time zone that prohibits live participation. Such students must describe such circumstances to the instructor in advance.

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- Start them early! :)

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- If two collaborators submit material that looks suspiciously similar, both will get lower marks

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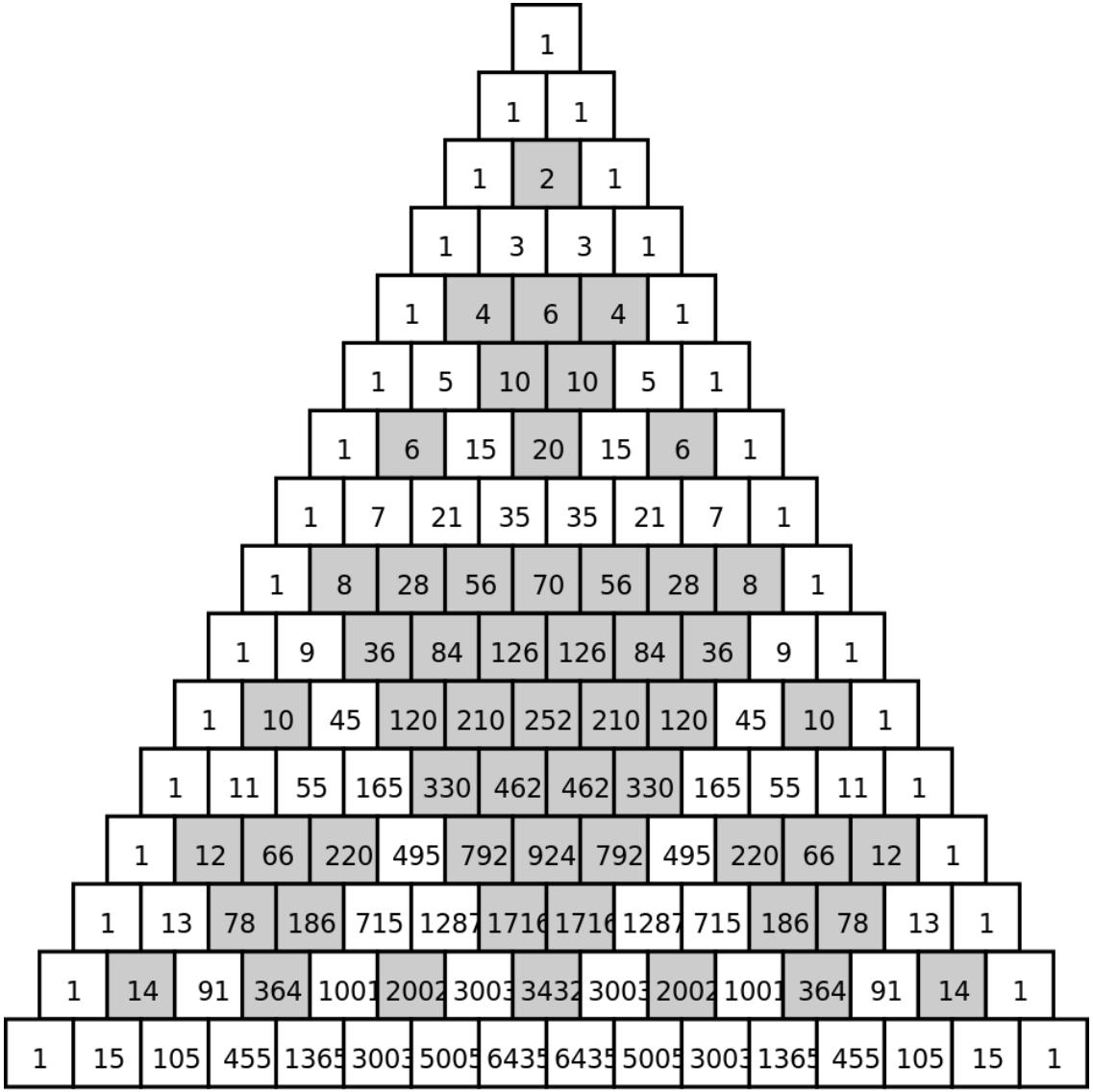
Example resources for learning LaTeX:

- `https://www.overleaf.com/learn/latex/Free_online_introduction_to_LaTeX_(part_1)`
- `https://artofproblemsolving.com/wiki/index.php/LaTeX`

Questions?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