COMP 761: Lecture 27 – Max Flow

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November 6, 2020

Problem

A *cut* in a flow network is a partition of the vertices into S and T, where $s \in S$ and $t \in T$. The *capacity* of the cut (S, T) is:

$$c(S, T) = \sum_{u \in S} \sum_{v \in T} c(u, v).$$

Prove that $|f| \leq c(S, T)$ for any flow f and cut (S, T).

(Please don't post your ideas in the chat just yet, we'll discuss the problem soon in class.)

Course Announcements

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- On Problem 1, reminder not just to cite that polynomials are Θ(leading term) unless you prove that. May want to provide a constant c for Ω notation explicitly. If this was not clear to you, feel free to take an additional 24 hours before submitting (you can edit an existing submission). Slack message with any questions.

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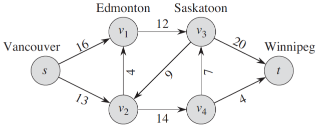
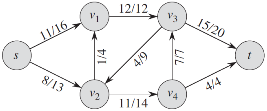
- We assume there are no self-loops (u, u).
- A *flow* from s to t is a function f where if (u, v) is an edge, then

$$f(u, v) \leq c(u, v)$$

and $f(u, v) = 0$ if there is no edge (u, v), and where for all u different from s and t,

$$\sum_{v} f(v, u) = \sum_{v} f(u, v).$$





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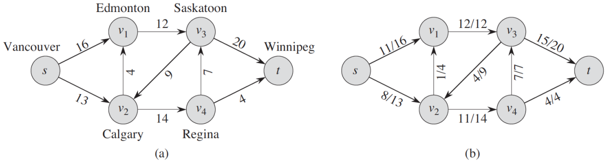

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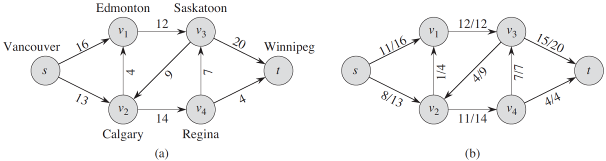

- We say that the *value* $|f|$ of a flow f is defined by:

$$|f| = \sum_v f(s, v) - \sum_v f(v, s).$$

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- (There might be several with this maximum value.)
- Applications include routing shipments, calculating flow of water or electricity, airplane scheduling, image segmentation, etc.

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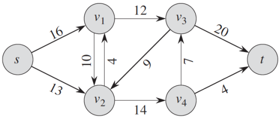
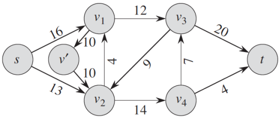


(a)



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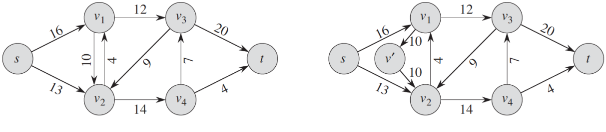

(a) (b)

- Note that we can't just "cancel" the antiparallel capacities, since that would prevent us traveling in one direction along that edge.

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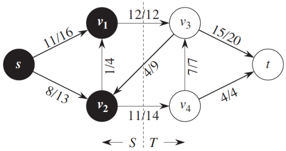

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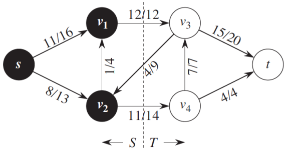

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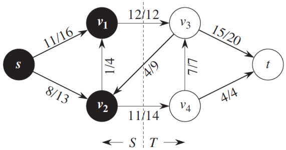

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$$= \sum_v f(s, v) - \sum_v f(v, s) + \sum_{u \in S - \{s\}} \left(\sum_v (f(u, v) - f(v, u)) \right)$$

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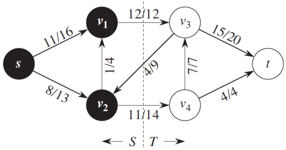

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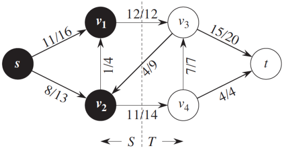

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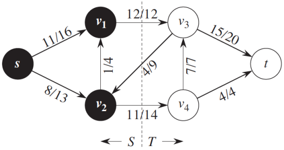

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- Why is this true?

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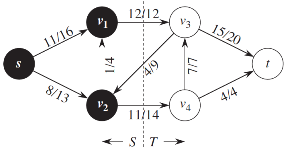

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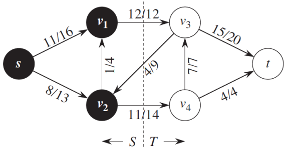

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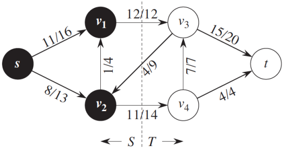

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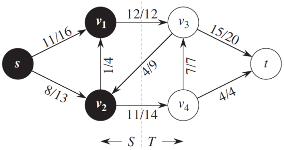

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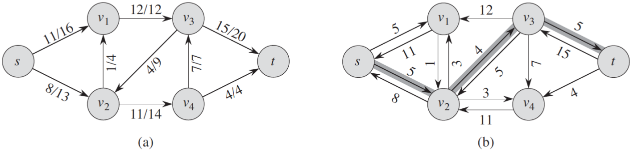
Residual network

- Given a flow network G and a flow f in it, we can define the *residual network* G_f as follows:
- If (u, v) is an edge in G, then both (u, v) and (v, u) are edges in G_f, with *residual capacities*

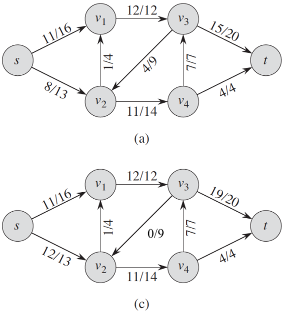
$$c_f(u, v) = c(u, v) - f(u, v)$$
$$c_f(v, u) = f(u, v).$$



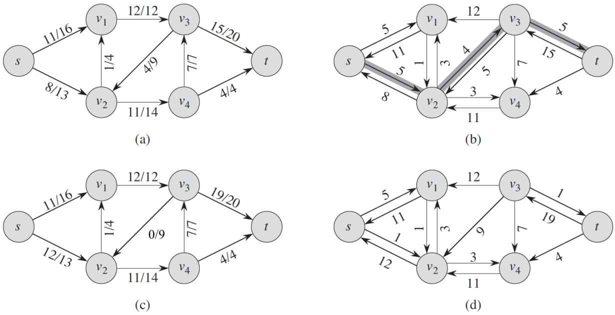
Augmenting paths

Augmenting paths

- Given a flow network G and a flow f from s to t, an *augmenting path* is a path from s to t in the residual network with positive capacity along all edges.



Ford-Fulkerson method



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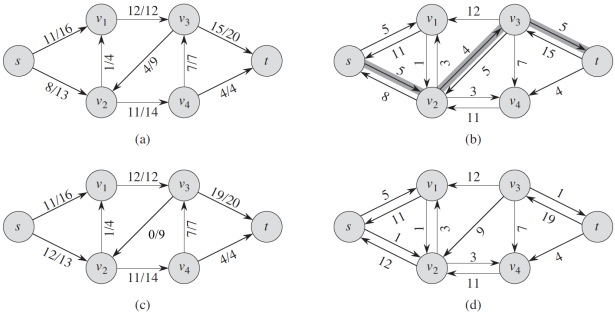

- The *Ford-Fulkerson method* for solving max flow consists of:
  1. Start out with $f(u, v) = 0$ everywhere.
  2. Find an augmenting path in the residual network G_f.
  3. Update f by adding flow along that path.
  4. Repeat Steps 2 and 3 until there is no augmenting path.

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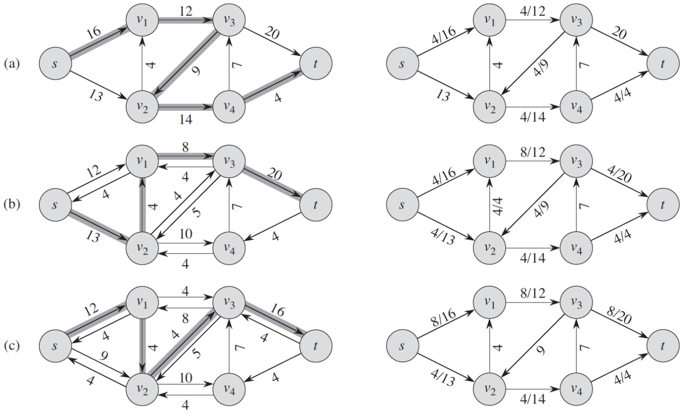

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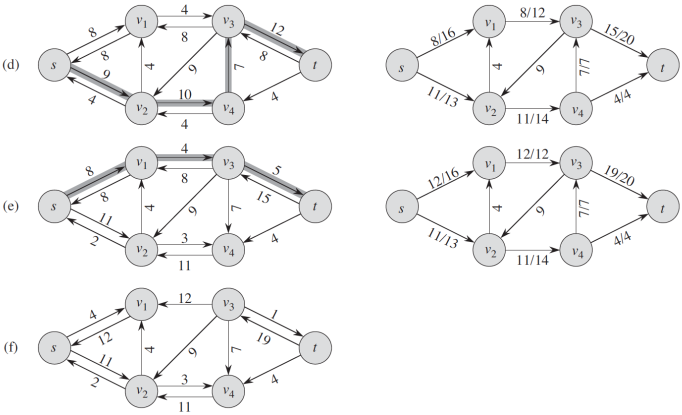

Max-flow min-cut theorem

Will prove Ford-Fulkerson by showing the following are all equivalent.

1. f is a maximum flow in G.
2. The residual network G_f contains no augmenting paths.
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This is called the **max-flow min-cut theorem**.

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- Why does (3) imply (1)?
- We know $|f| \leq c(S, T)$ for every flow f, so if $|f| = c(S, T)$, then f must be maximum.

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- Let S be the set of vertices u where there's a path s to u in G_f.

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- This shows that the Ford-Fulkerson method will eventually give us a max flow.

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- An improvement in the algorithm (the Edmonds-Karp algorithm) can actually make it:

$$O(|V| \cdot |E|^2).$$